MAHIDOL UNIVERSITY
Wisdom of the Land

[SCPY384]

Geophysical Prospecting

Class 2: 28 JAN 2019

Content: Direct-current resistivity survey / Instrument, data and examples

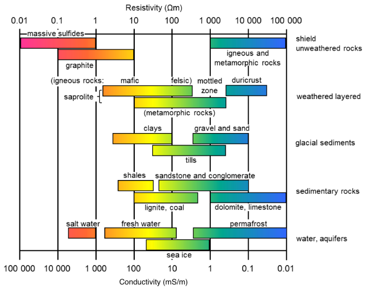
Instructor: Puwis Amatyakul

Today's Goals

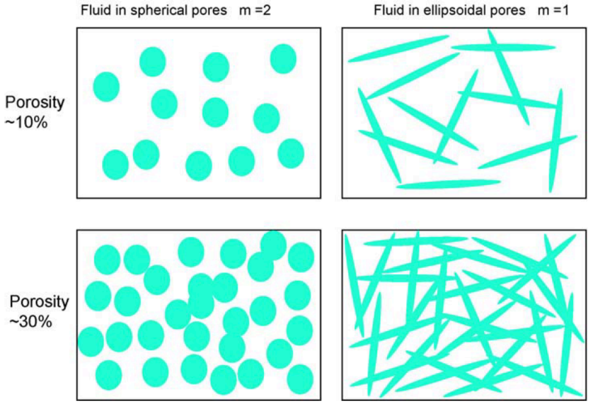
Part I: Reviews
Part II: DCR principle
Part III: DCR Instruments

Reviews

1. Resistance VS Resistivity



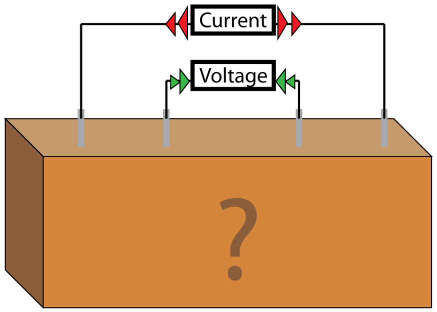
Physical interpretation of the cementation factor, m



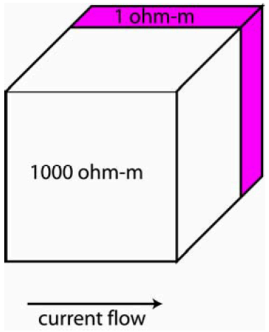
2. Archie's Law

$$\frac{\rho_o}{\rho_w} = F = \phi^{-m}$$

3. Direct-current resistivity survey



A – Fluid in cracks parallel to electric current flow



The sample has 10% porosity. This fluid geometry represents a **parallel circuit**, and electric current can effectively bypass the resistive rock grains and travel through the sample entirely in the conductive liquid.

What is the overall resistivity of the cube?

Today’s Goals

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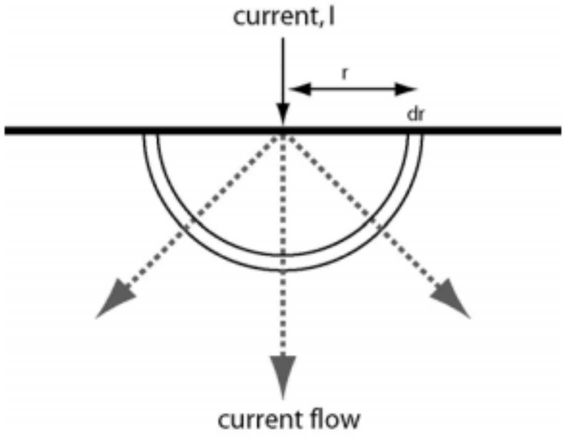
Part II: DCR principle
Part III: DCR Instruments

DC Resistivity Principle

1.1 Potential of a single current electrode



Consider an electric current, I, flowing from an electrode. The air has a very high electrical resistivity, so all current flows in the Earth. From symmetry arguments, the current spreads out uniformly in all directions. Now consider a shell of rock, with radius, *r*, and thickness *dr*. The voltage (potential) drop across the shell is ΔV

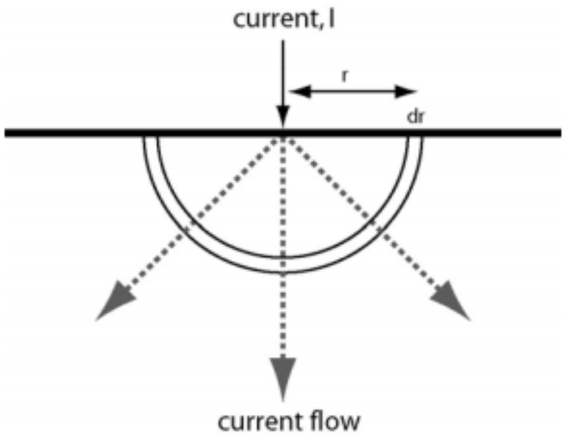
The resistance of the hemispherical shell, $\mathrm{R} = \frac{\rho L}{A} = \frac{\rho dr}{2\pi r^2} = \frac{\Delta V}{I}$

Rearranging and taking limits gives $dV = \frac{I\rho}{2\pi r^2} dr$

Source: https://sites.ualberta.ca/~unsworth/UA-classes/325/C/325C2-2005.pdf

DC Resistivity Principle

1.1 Potential of a single current electrode



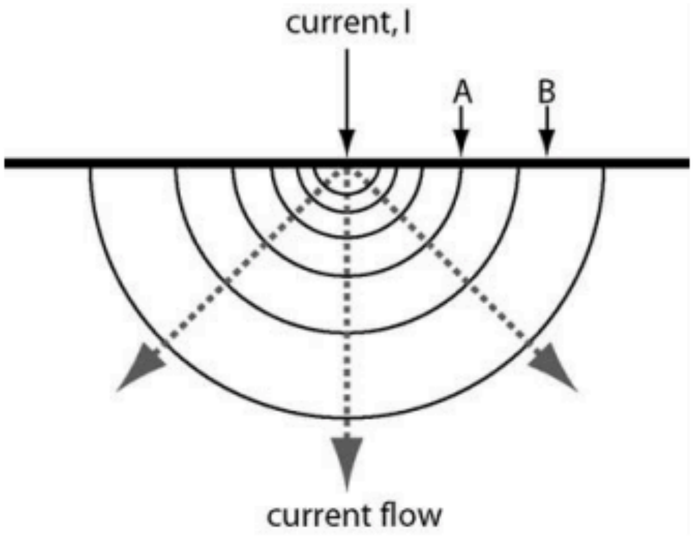
To compute the potential, V, apply the boundary condition that V = 0 when r = ∞ and integrate to give:

$$V = \frac{I\rho}{2\pi}\int\frac{1}{r^2}dr = \frac{I\rho}{2\pi}\left[-\frac{1}{r}\right]_r^\infty = -\frac{I\rho}{2\pi r}$$

DC Resistivity Principle

1.1 Potential of a single current electrode

Can this geometry be used to measure the resistivity of the Earth?



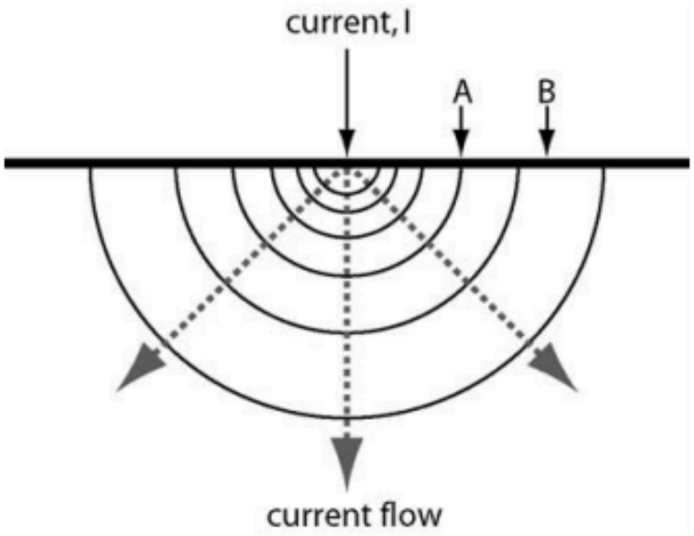
The voltage between the electrodes A and B is defined as $\Delta V_{AB} = V_A - V_B$

Using the above result $\Delta V_{AB} = \frac{I\rho}{2\pi}\left[\frac{1}{r_A} - \frac{1}{r_B}\right]$

DC Resistivity Principle

1.1 Potential of a single current electrode

Can this geometry be used to measure the resistivity of the Earth?



$$\Delta V_{AB} = \frac{I\rho}{2\pi}\left[\frac{1}{r_A} - \frac{1}{r_B}\right]$$

where r_A and r_B are the distances from the current electrode to the potential electrodes A and B respectively. Rearranging this equation gives

$$\rho = \frac{2\pi\Delta V_{AB}}{I(\frac{1}{r_A} - \frac{1}{r_B})}$$

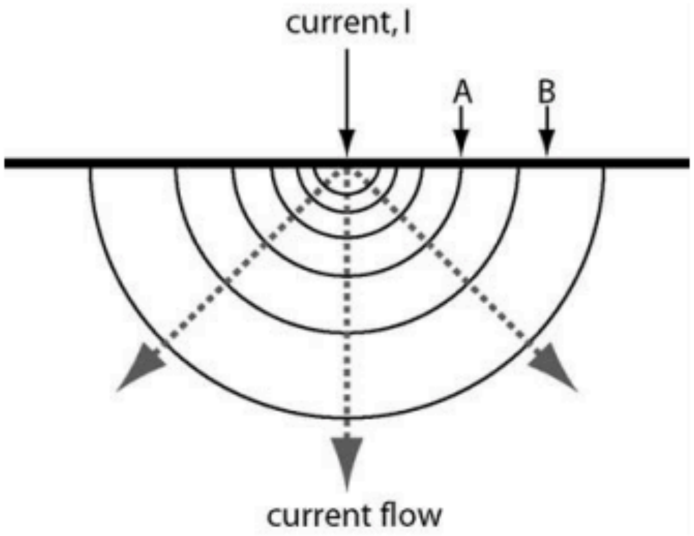
Note that this is essentially Ohms Law with a geometric factor added.

Why is this not a practical way to measure the resistivity of the Earth?

DC Resistivity Principle

1.1 Potential of a single current electrode

Can this geometry be used to measure the resistivity of the Earth?

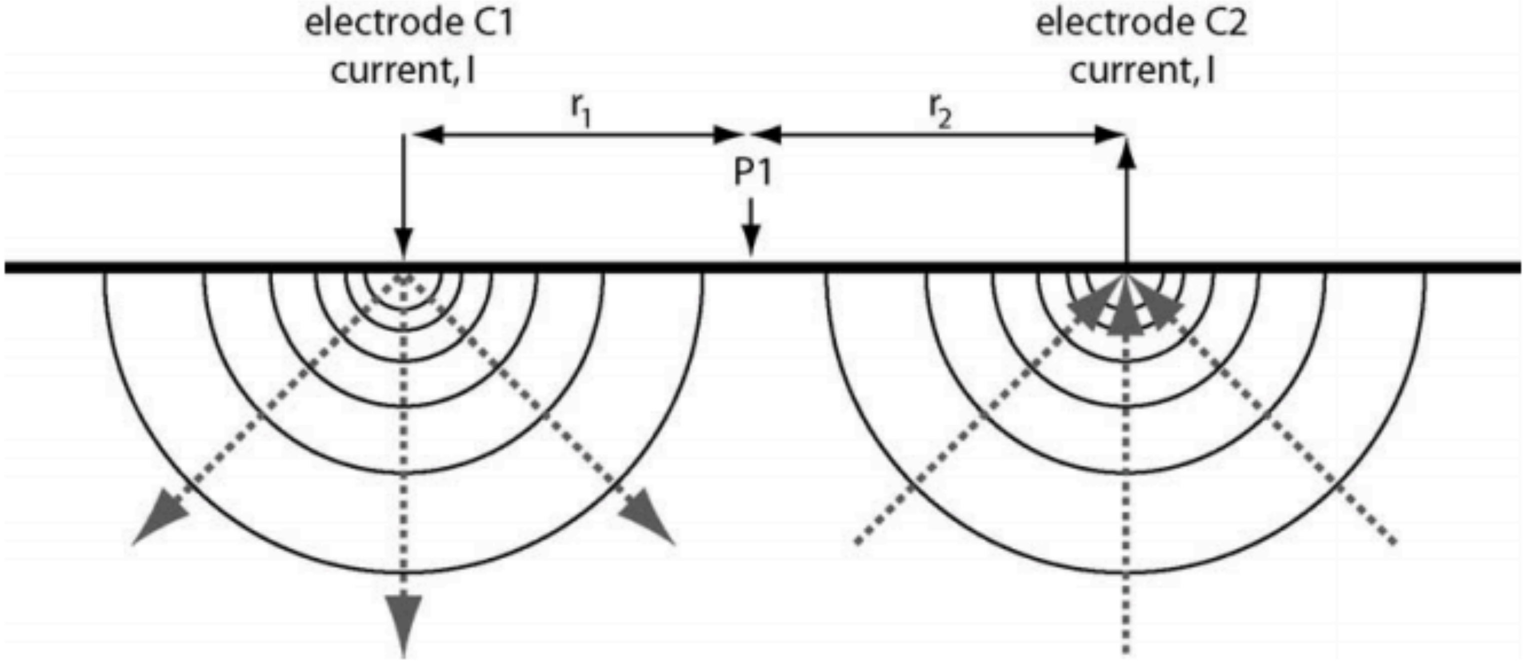


$$\boxed{\Delta V_{AB} = \frac{I\rho}{2\pi}\left[\frac{1}{r_A} - \frac{1}{r_B}\right]} \Rightarrow \rho = \frac{2\pi\Delta V_{AB}}{I\left(\frac{1}{r_A} - \frac{1}{r_B}\right)}$$

Why is this not a practical way to measure the resistivity of the Earth?

DC Resistivity Principle

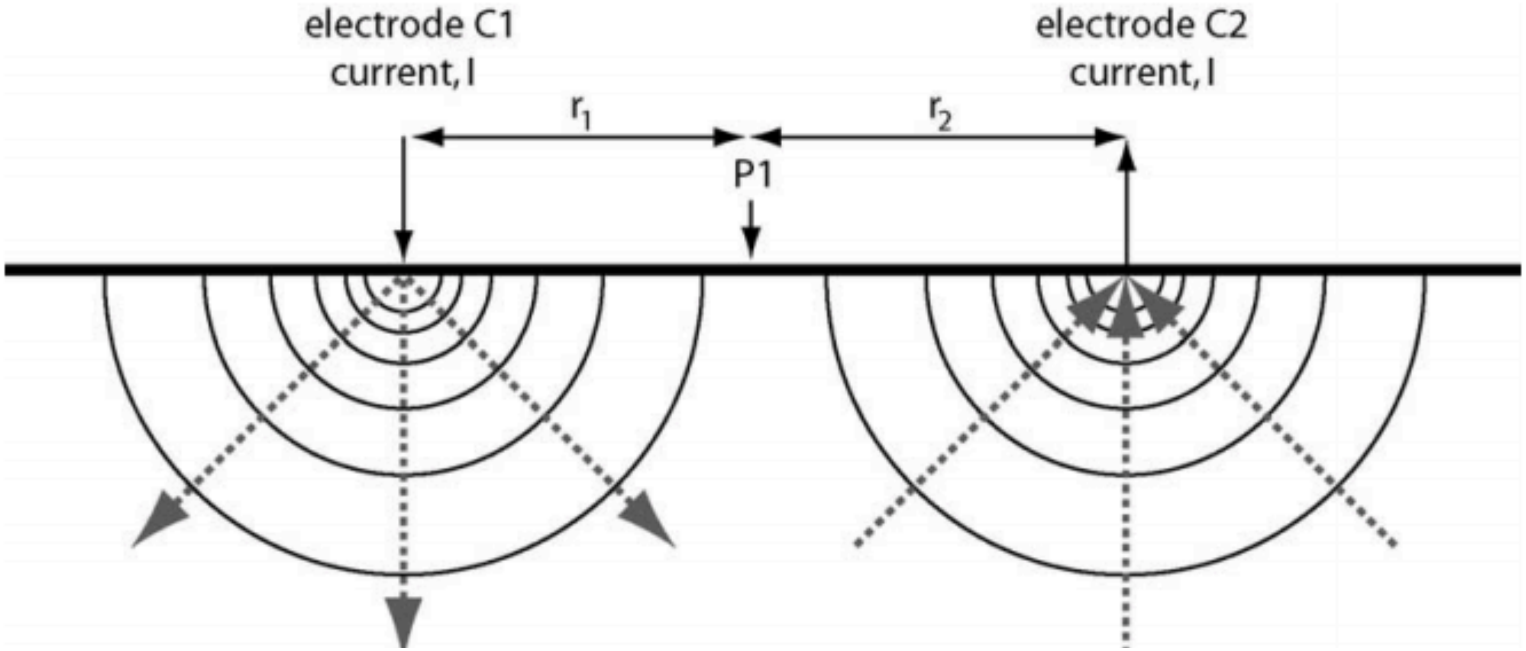
1.2 Potential of two current electrodes



A more realistic situation uses to current electrodes. Current is injected through one electrode and withdrawn through the other.

DC Resistivity Principle

1.2 Potential of two current electrodes

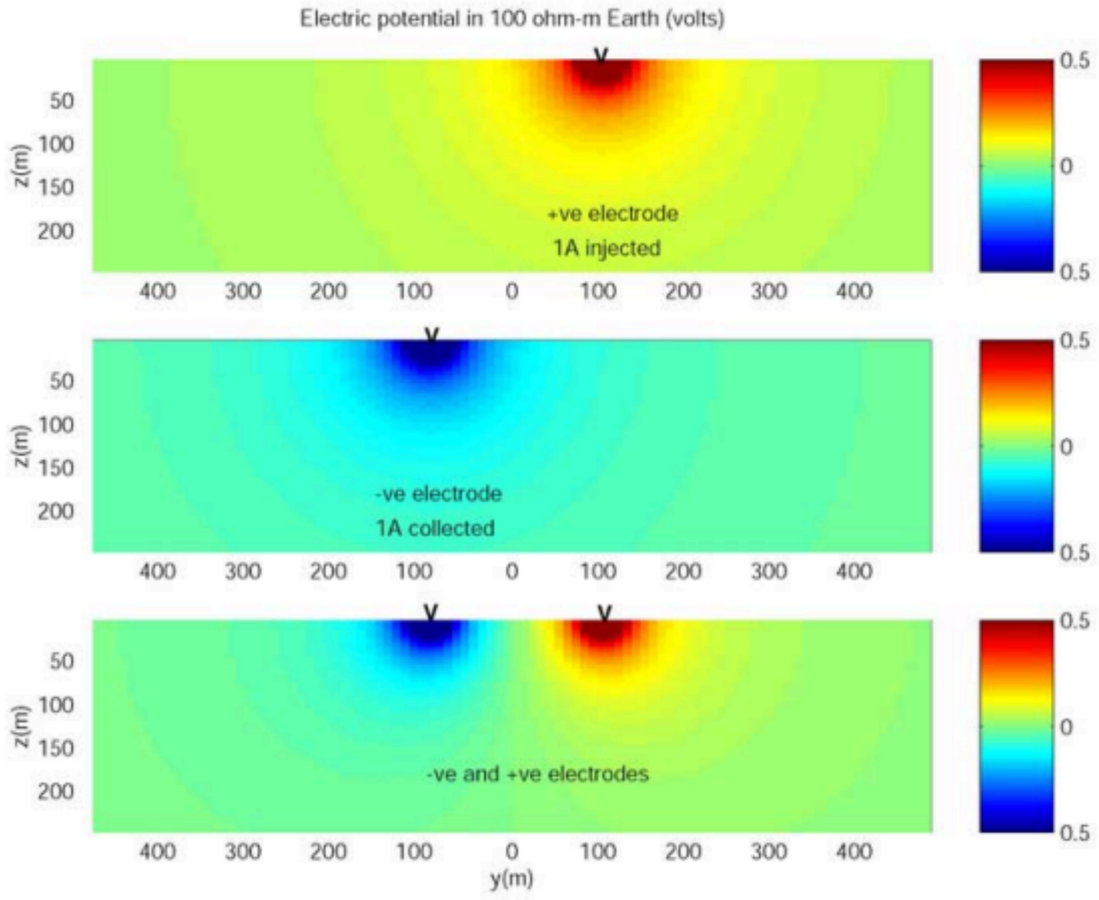


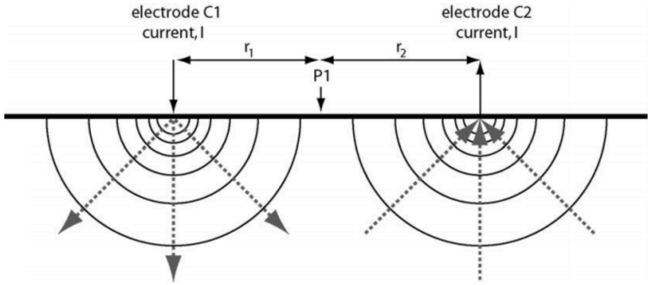
To compute the potential at electrode P1, we can simply add the potentials generated by the two current electrodes C1 and C2.

$$V_{P1} = \frac{I\rho}{2\pi r_1} - \frac{I\rho}{2\pi r_2} = \frac{I\rho}{2\pi}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

DC Resistivity Principle

1.2 Potential of two current electrodes

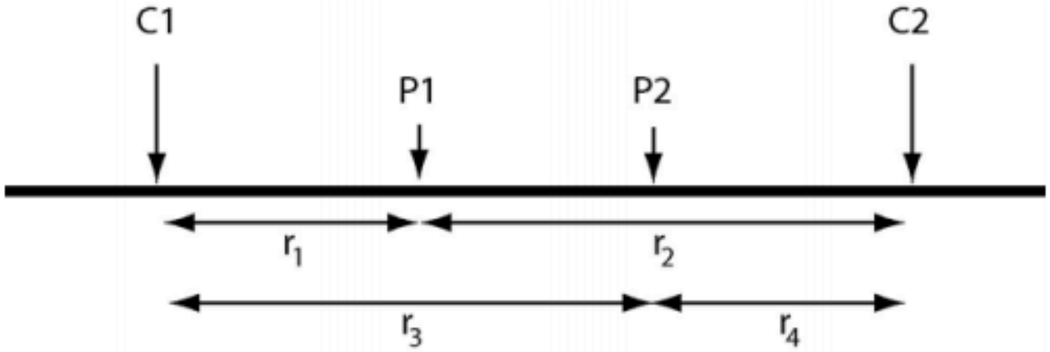




DC Resistivity Principle

1.2 Potential of two current electrodes

However, to measure a voltage, we need **two** potential electrodes to connect to a voltmeter. Consider the arrangement of electrodes shown below.



Can you prove that

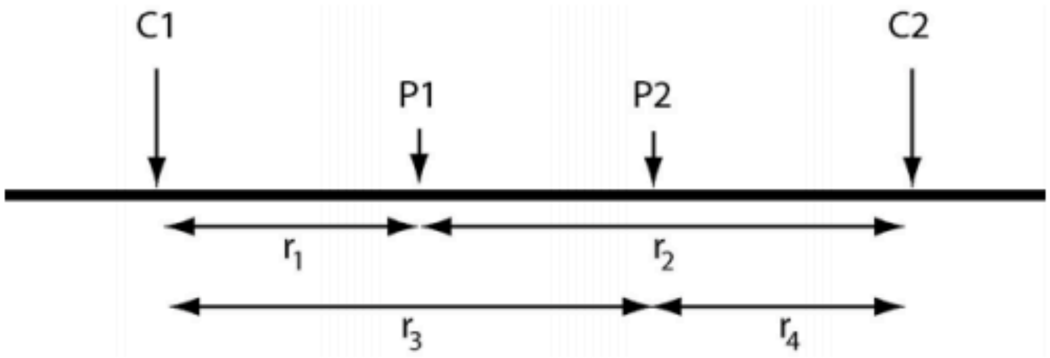
$$\Delta V = \frac{I\rho}{2\pi}\left[\frac{1}{r_1} - \frac{1}{r_2} - \frac{1}{r_3} + \frac{1}{r_4}\right]$$

General case

DC Resistivity Principle

1.2 Potential of two current electrodes

However, to measure a voltage, we need **two** potential electrodes to connect to a voltmeter. Consider the arrangement of electrodes shown below.



Special case

Now let us make the geometry of the array simple, with the 4 electrodes separated by a distance *a*. Then we have $r_1 = r_4 = a$ and have $r_3 = r_3 = 2a$

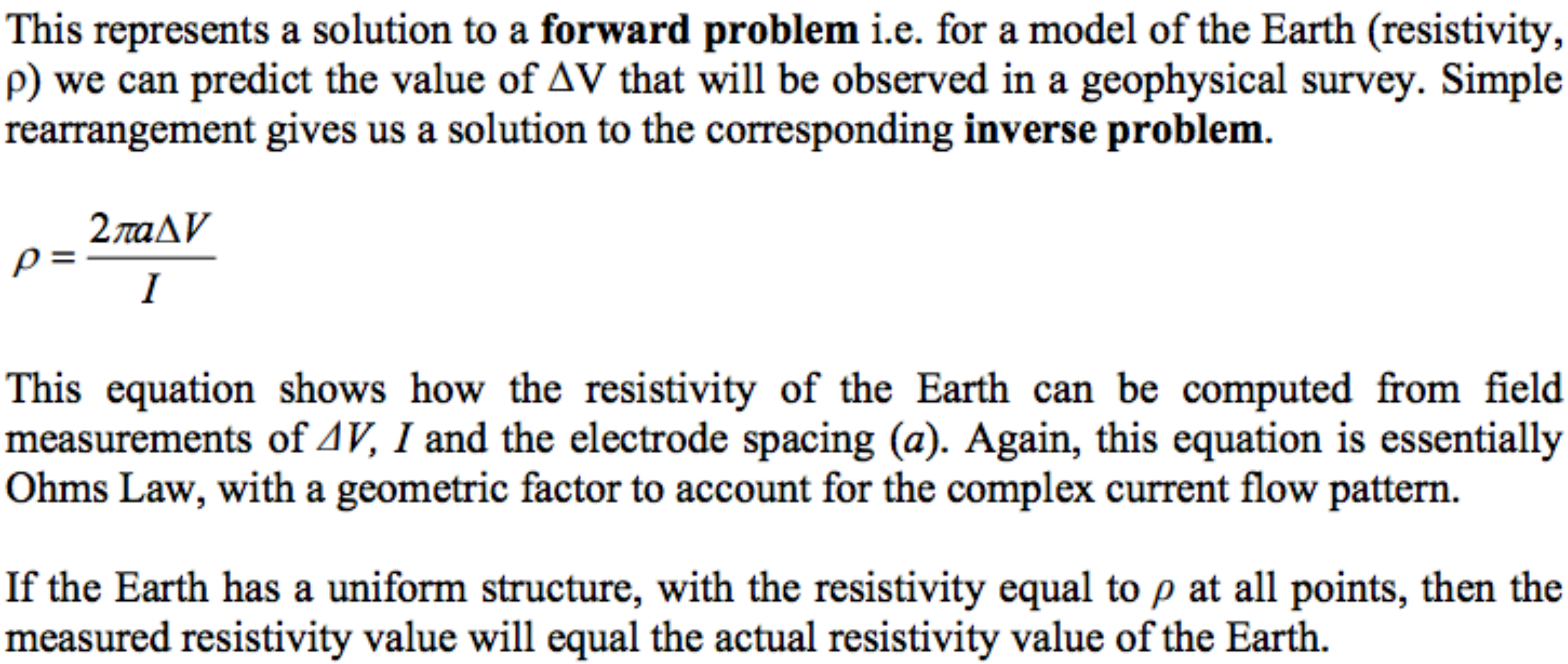
$$\Delta V = V_{P1} - V_{P2} = \frac{I\rho}{2\pi}\left[\left(\frac{1}{a} - \frac{1}{2a}\right) - \left(\frac{1}{2a} - \frac{1}{a}\right)\right]$$

$$= \frac{I\rho}{2\pi a}\left[\left(1 - \frac{1}{2}\right) - \left(\frac{1}{2} - 1\right)\right]$$

$$= \frac{I\rho}{2\pi a}$$

DC Resistivity Principle

1.2 Potential of two current electrodes



This represents a solution to a **forward problem** i.e. for a model of the Earth (resistivity, ρ) we can predict the value of ΔV that will be observed in a geophysical survey. Simple rearrangement gives us a solution to the corresponding **inverse problem**.

$$\rho = \frac{2\pi a \Delta V}{I}$$

This equation shows how the resistivity of the Earth can be computed from field measurements of ΔV, I and the electrode spacing (a). Again, this equation is essentially Ohms Law, with a geometric factor to account for the complex current flow pattern.

If the Earth has a uniform structure, with the resistivity equal to ρ at all points, then the measured resistivity value will equal the actual resistivity value of the Earth.

DC Resistivity Principle

1.2 Potential of two current electrodes

However, if the resistivity is variable, the resistivity computed will be an average value over the region in which the current is flowing. This average resistivity is termed the **apparent resistivity** and defined as

$$\rho_a = \frac{2\pi a \Delta V}{I}$$

The figure below shows a quantitative evaluation of the electric current flow pattern.

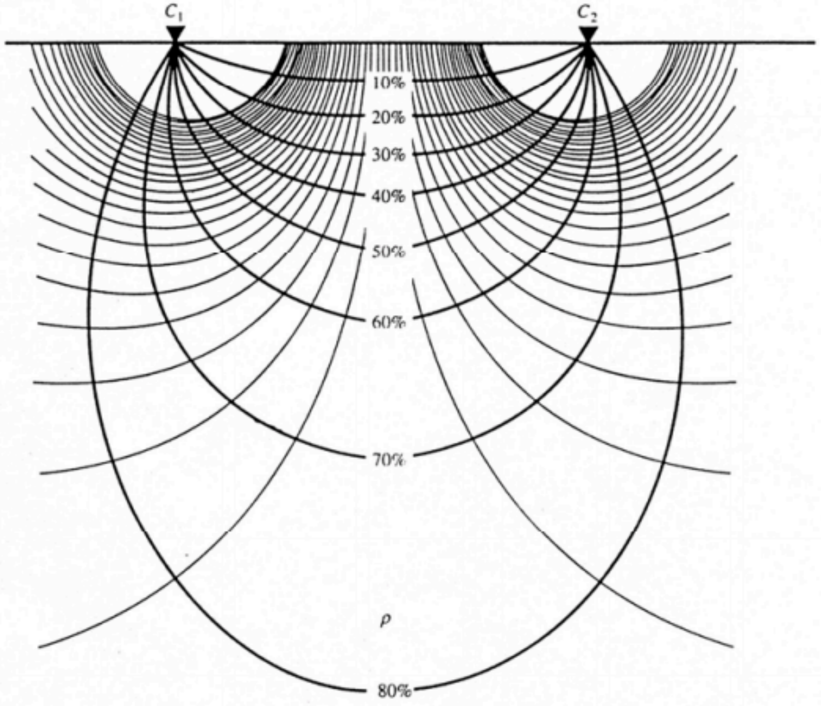


Figure 5-8 Equipotential surfaces and current lines of flow. Labels indicate percent of total current that penetrates to the depth of the line.

DC Resistivity Principle

1.2 Potential of two current electrodes

The figure below shows a quantitative evaluation of the electric current flow pattern.

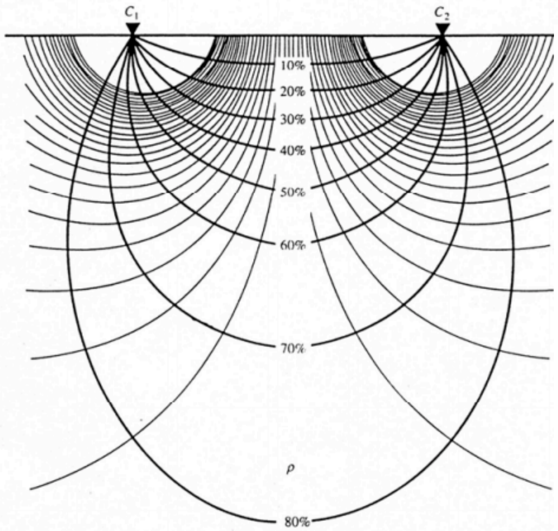


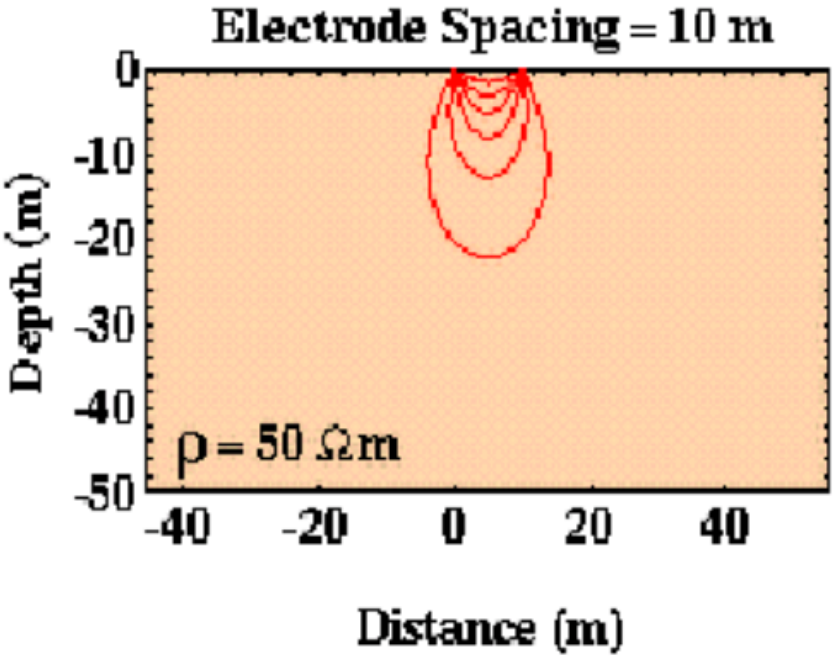
Figure 5-8 Equipotential surfaces and current lines of flow. Labels indicate percent
total current that penetrates to the depth of the line.

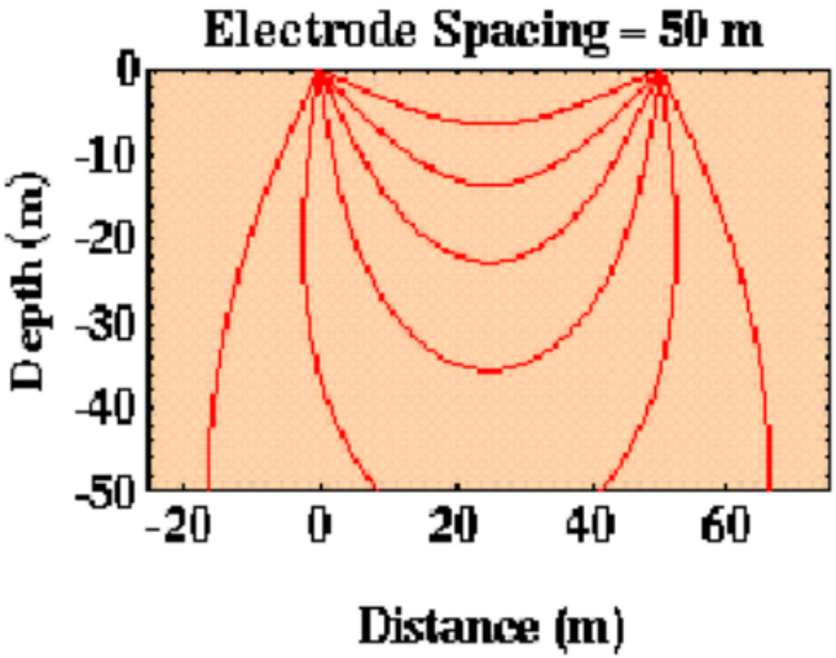
Note that:

- Electric current does not flow directly from one current electrode to the other in a straight line. This is because the charge carriers repel one another.
- Electric current flow is at right angles to the equipotential surfaces.
- Approximately 50% of the electric current flows within a depth *a* of the surface.
- The apparent resistivity can be considered the average resistivity over a volume that is located between the electrodes, and in the depth range from the surface to a depth equal to the *a*-spacing.

DC Resistivity Principle

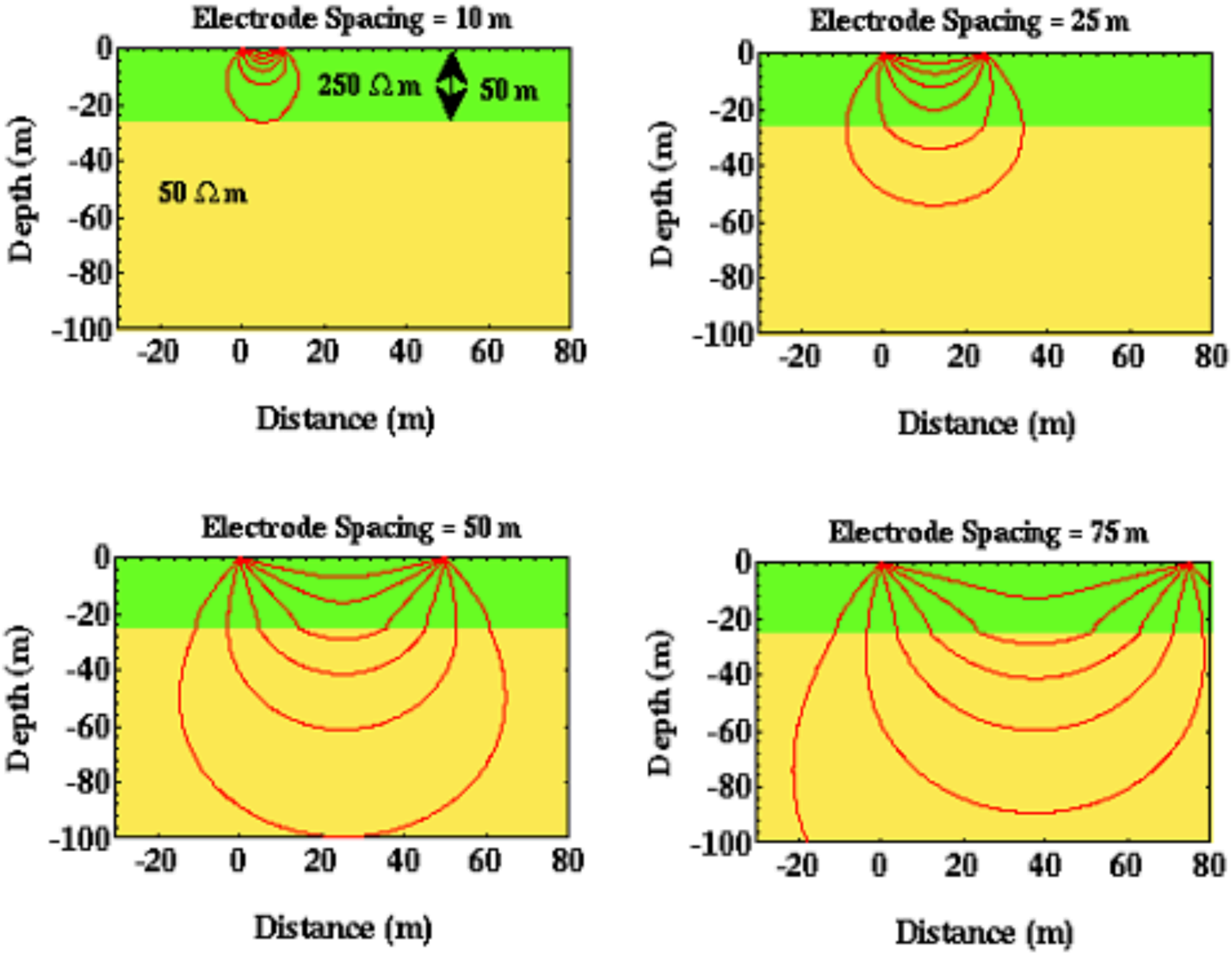
1.2 Complex underground resistivity





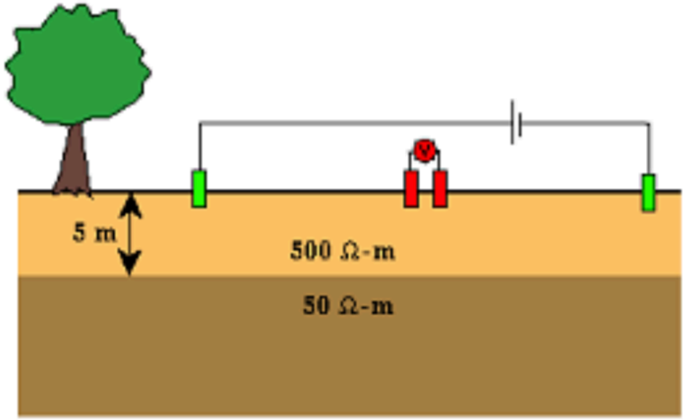
DC Resistivity Principle

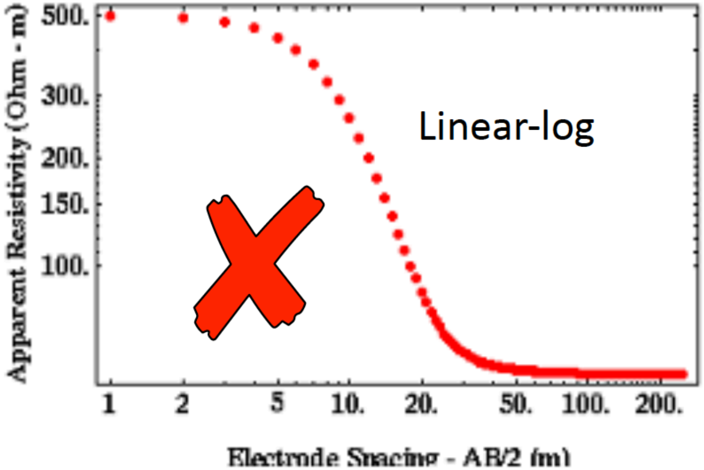
1.2 Complex underground resistivity

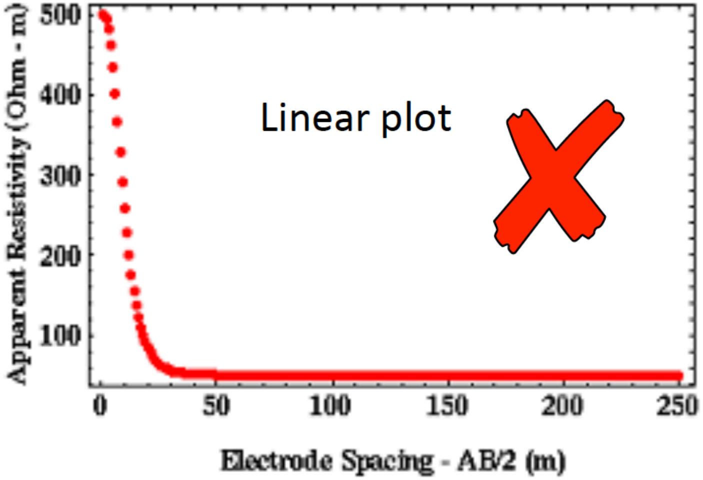


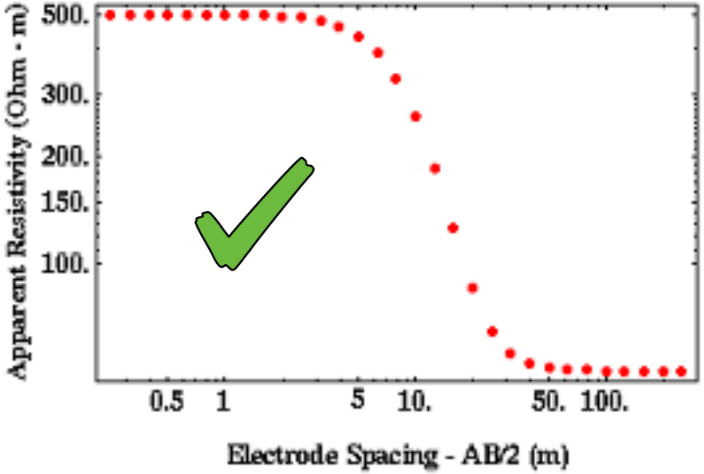
DC Resistivity Principle

1.2 Complex underground resistivity





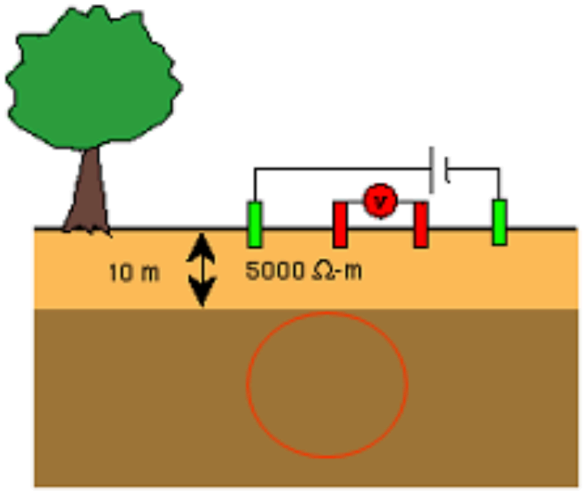


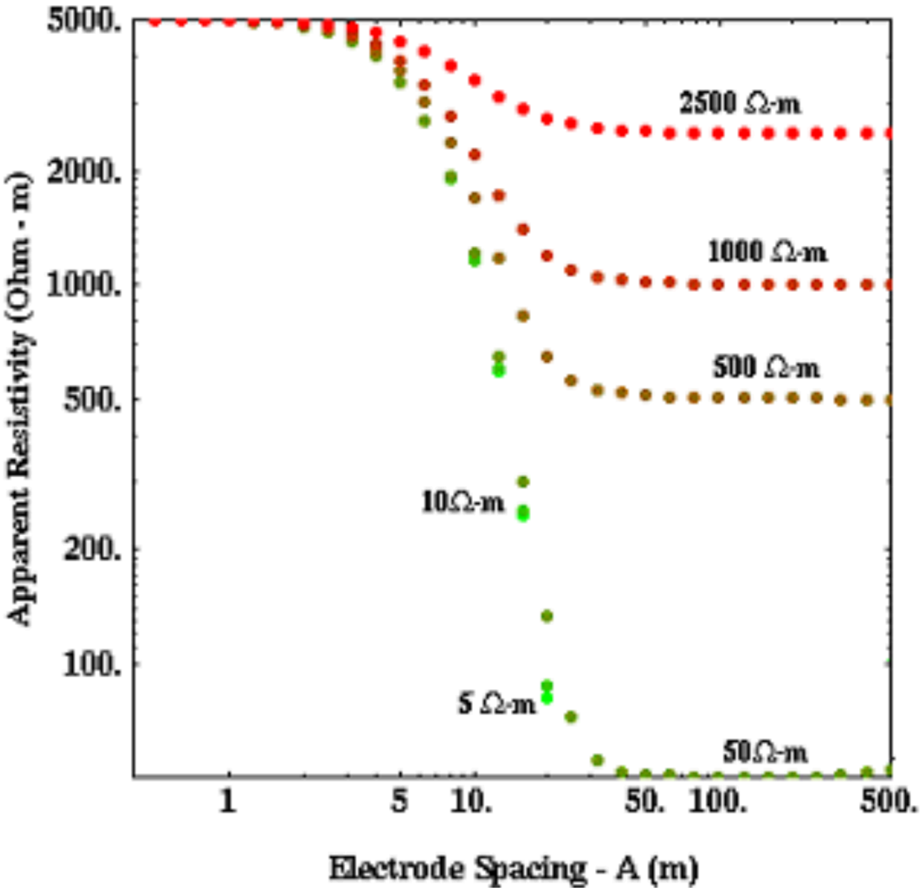


DC Resistivity Principle

1.2 Complex underground resistivity

Apparent Resistivity Curve
Two-layered media –case 1

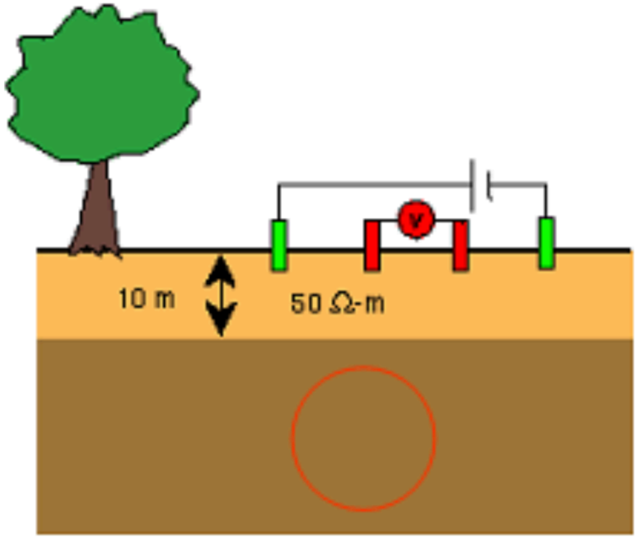


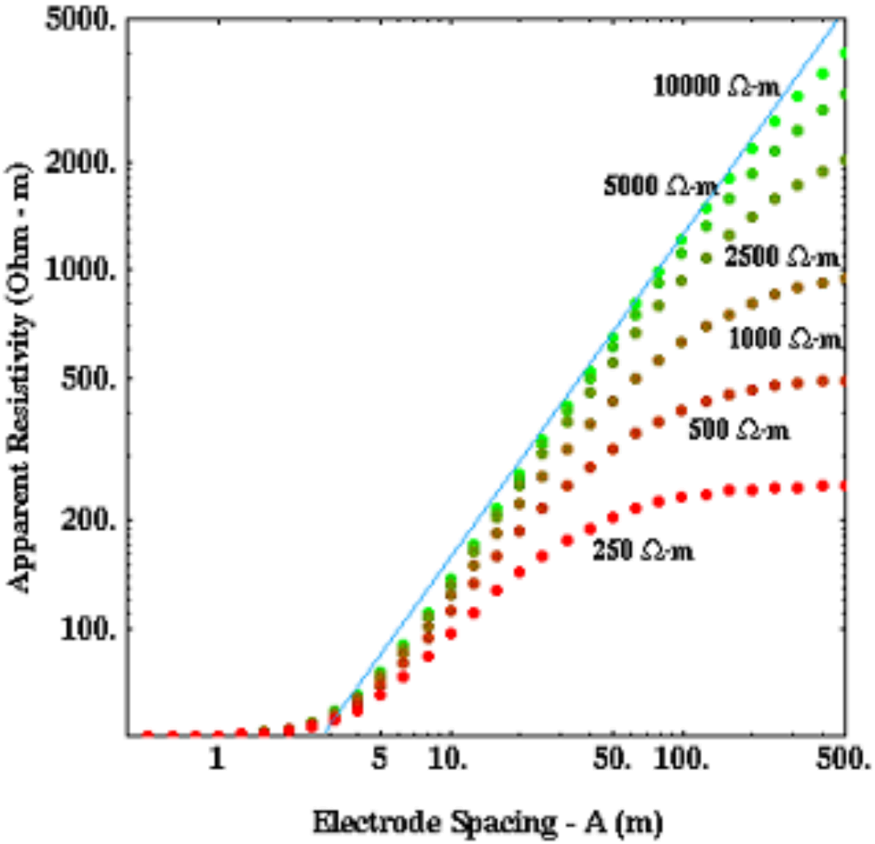


DC Resistivity Principle

1.2 Complex underground resistivity

Apparent Resistivity Curve
Two-layered media –case 2

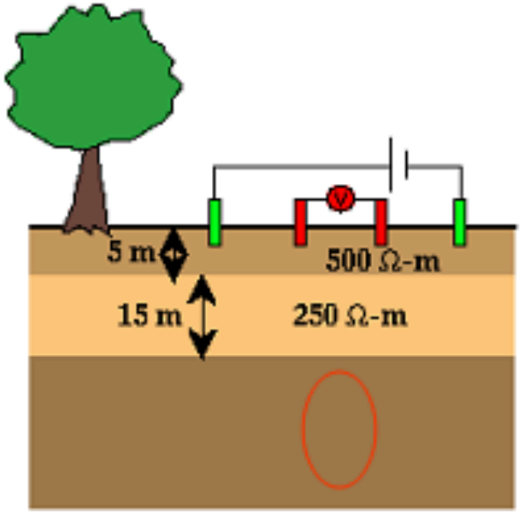


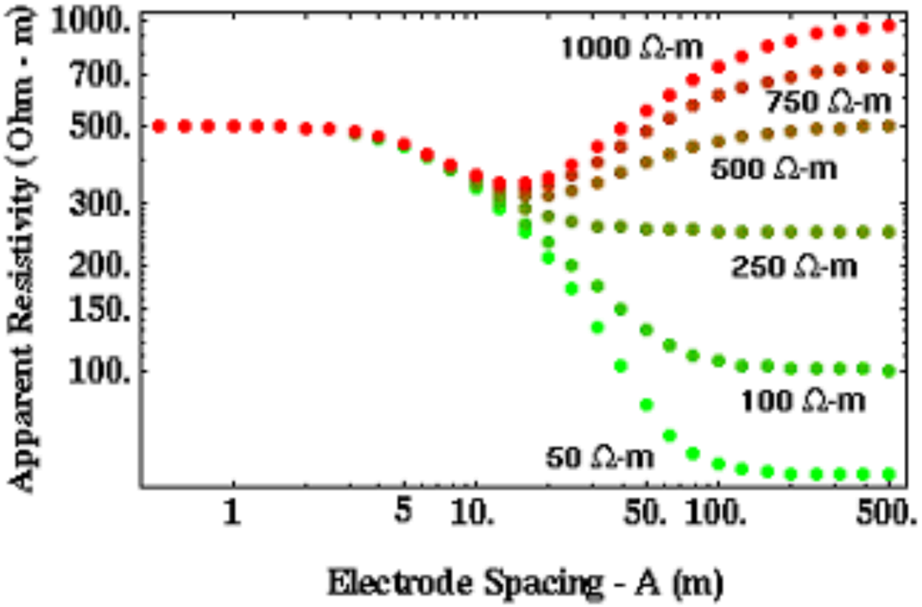


DC Resistivity Principle

1.2 Complex underground resistivity

Apparent Resistivity Curve
Three-layered media

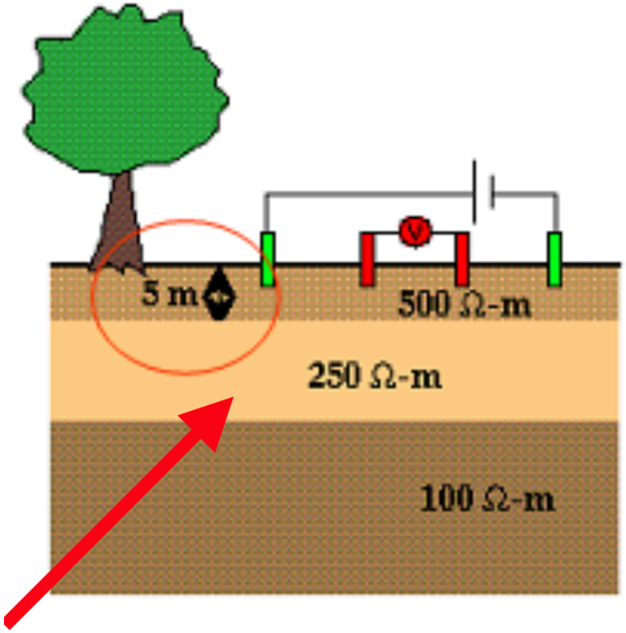


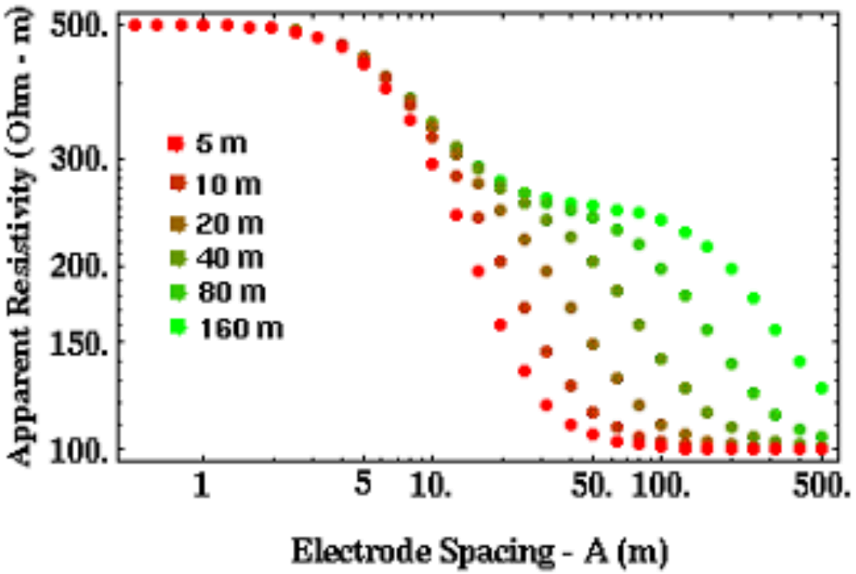


DC Resistivity Principle

1.2 Complex underground resistivity

Apparent Resistivity Curve
Three-layered media





Today’s Goals

Part I: Reviews
Part II: DCR principle
Part III: DCR Instruments

DC Resistivity Measurement

Equipment

- Current source
- Ammeter
- Voltmeter
- Electrodes
- Cables

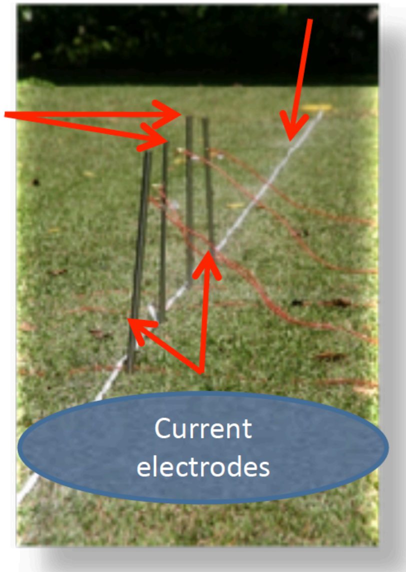
Hammer

Battery

Resisitivity Meter
SYSCAL Switch
(V11.5++),
IRIS
INSTRUMENTS

Measuring Tapes

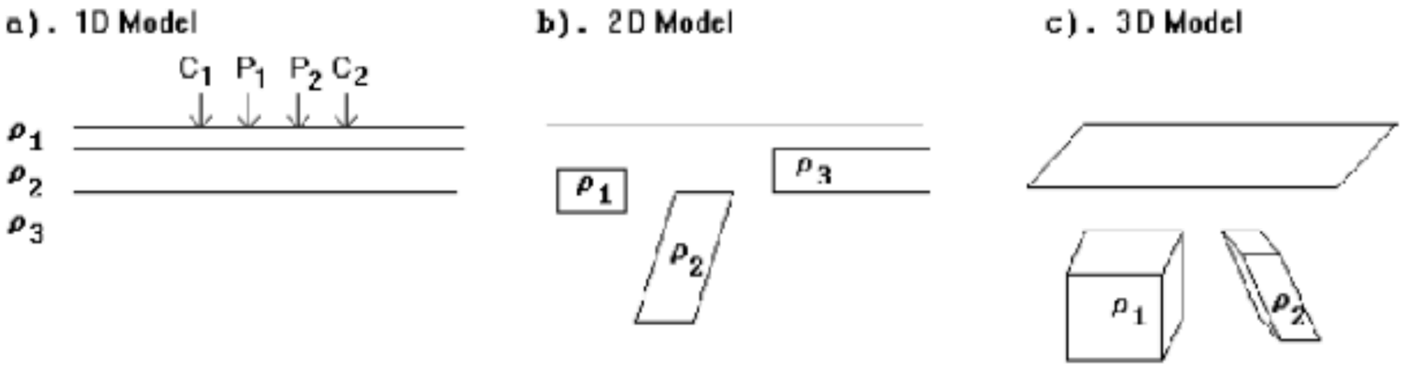
Potential Electrodes



DC Resistivity Measurement

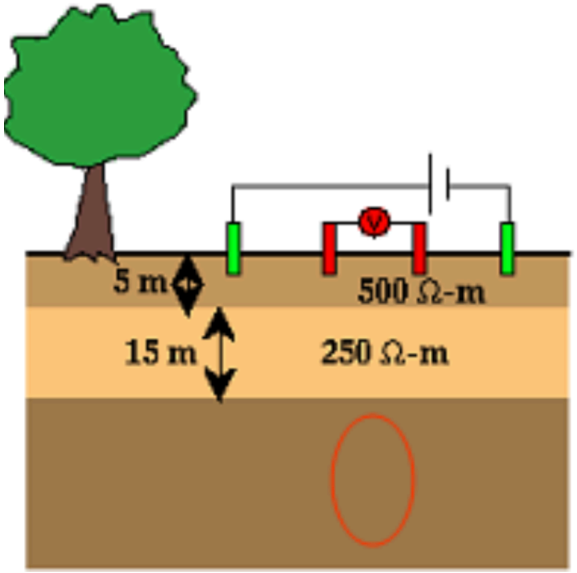
วิธีการสำรวจ

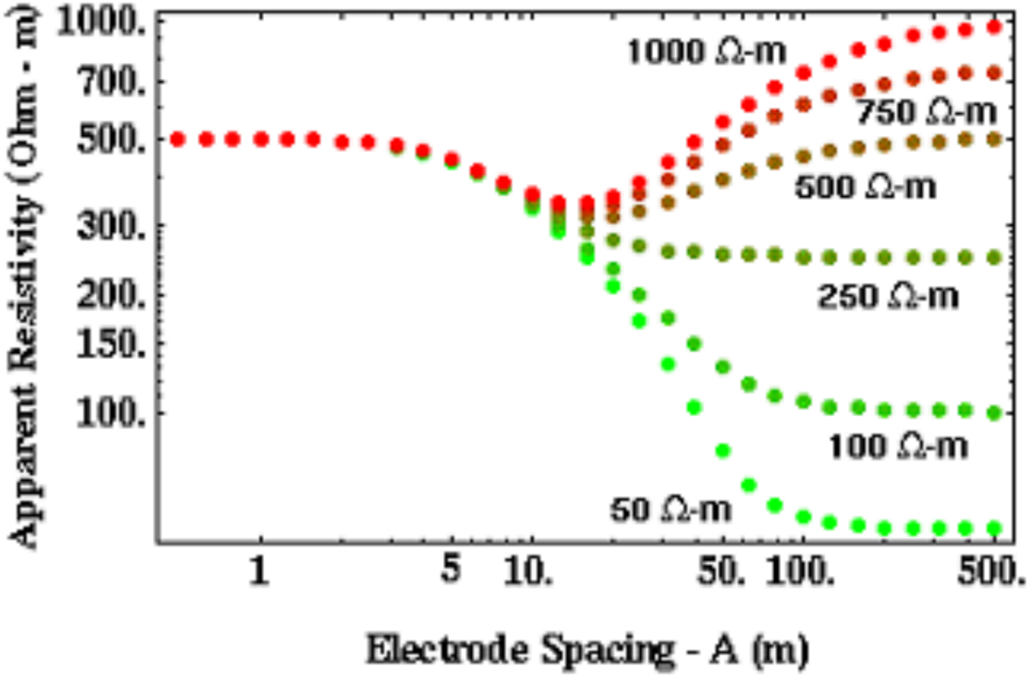
- 1-D : Vertical Electric Sounding (VES)
- 2-D : Electrical Profile
- 2-D : Electrical Resistivity Tomography (ERT)
- 3-D : ERT (series of 2-D ERTs)



DC Resistivity Measurement

- **1-D : Vertical Electric Sounding (VES)**
  - To look for variations in resistivity with depth

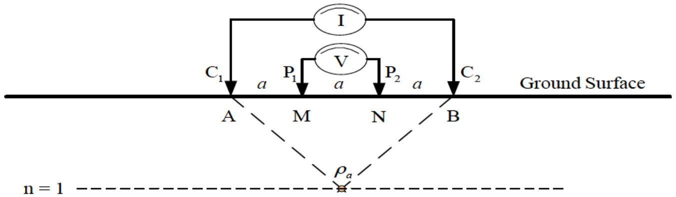




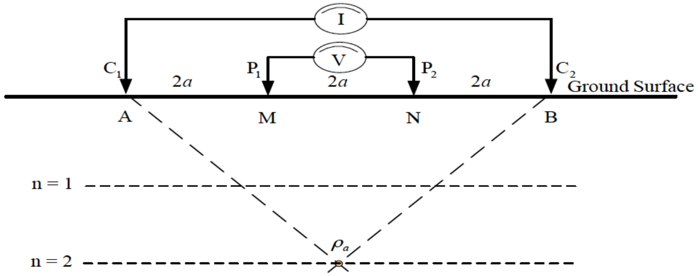
DC Resistivity Measurement

การขยายขั้วไฟฟ้าสำหรับ Wenner array

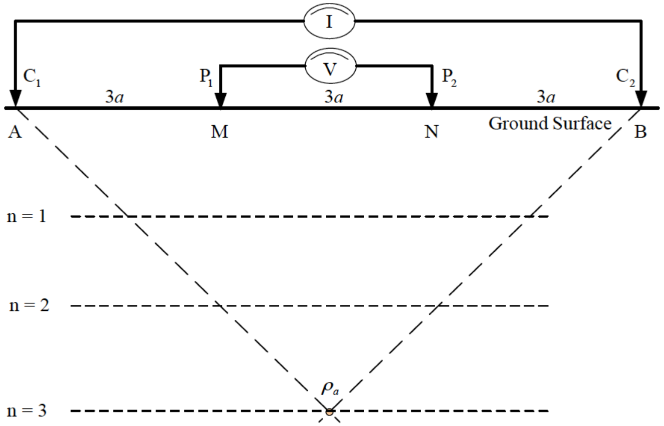
(a) n = 1



(b) n = 2

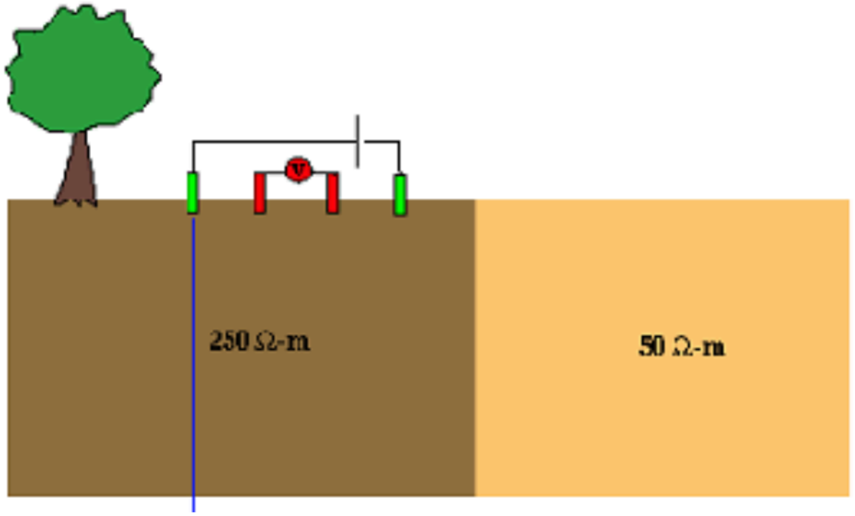


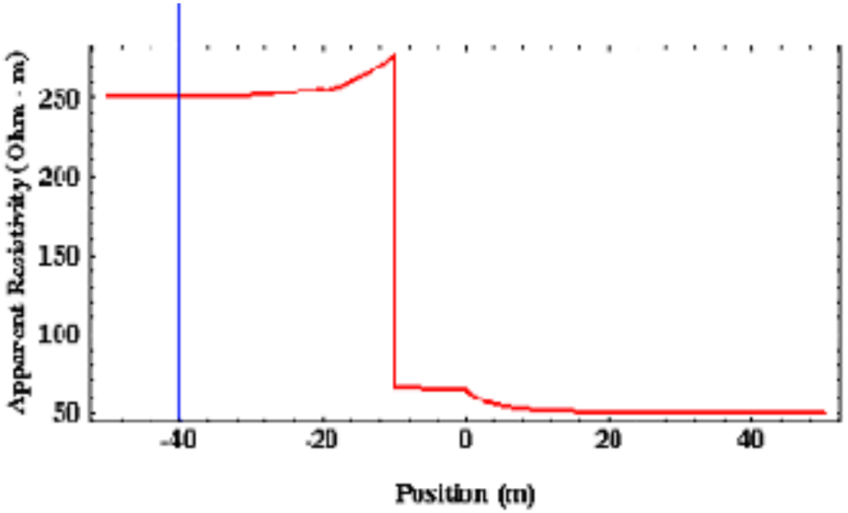
(c) n = 3



DC Resistivity Measurement

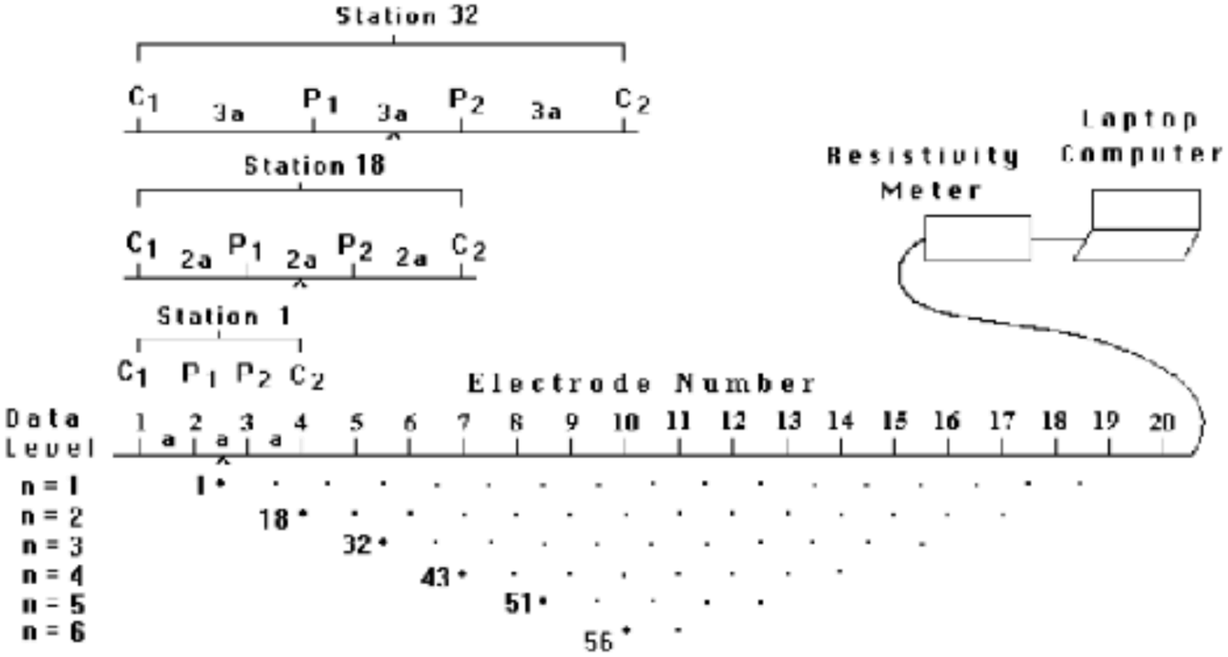
- **2-D : Electrical Profile**
  - To detect lateral variations in resistivity



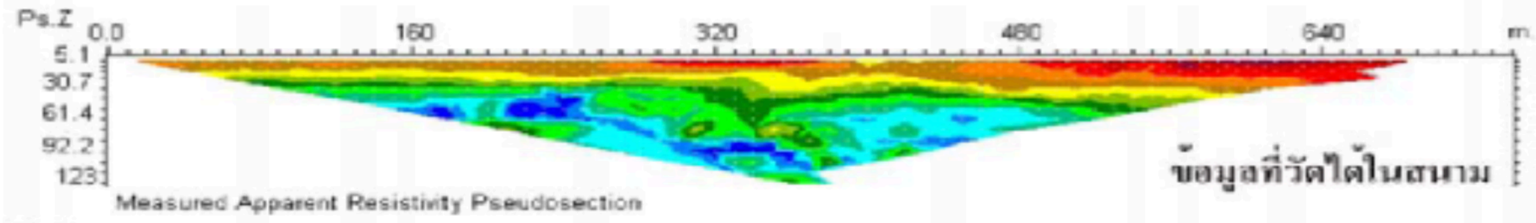


DC Resistivity Measurement

Pseudo-section

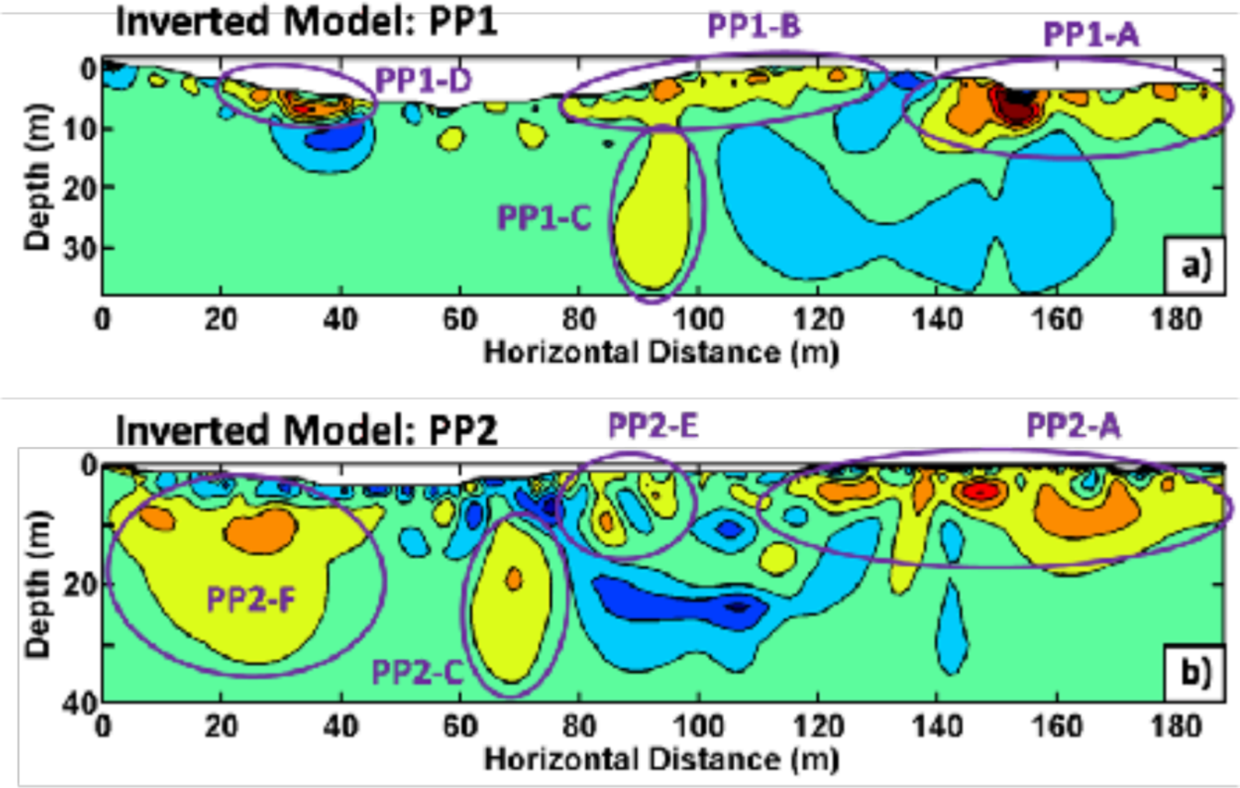


Sequence of measurements to build up a pseudosection



DC Resistivity Measurement

3-D : ERT (series of 2-D ERTs)



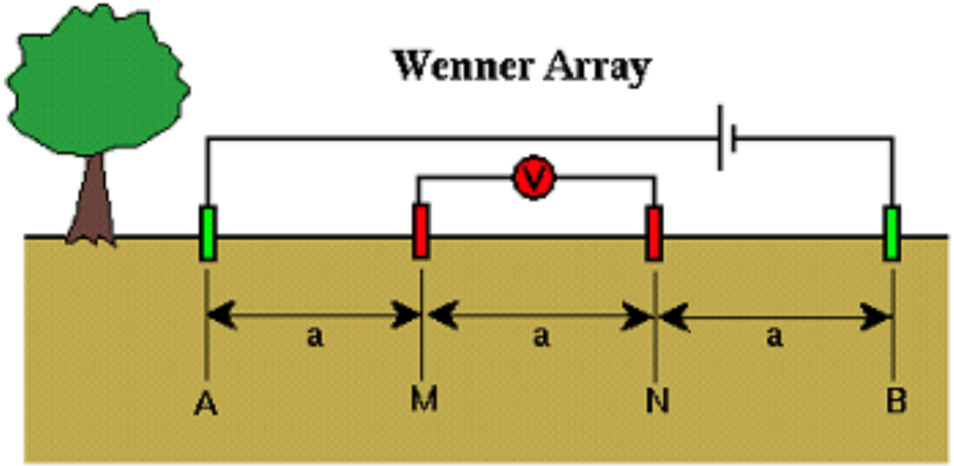
DC Resistivity Measurement

Array Type (Configuration)

- Pole-Pole array
- Pole-Dipole array
- Dipole-Dipole array
- Wenner array
- Schlumberger array

DC Resistivity Measurement

Wenner Array

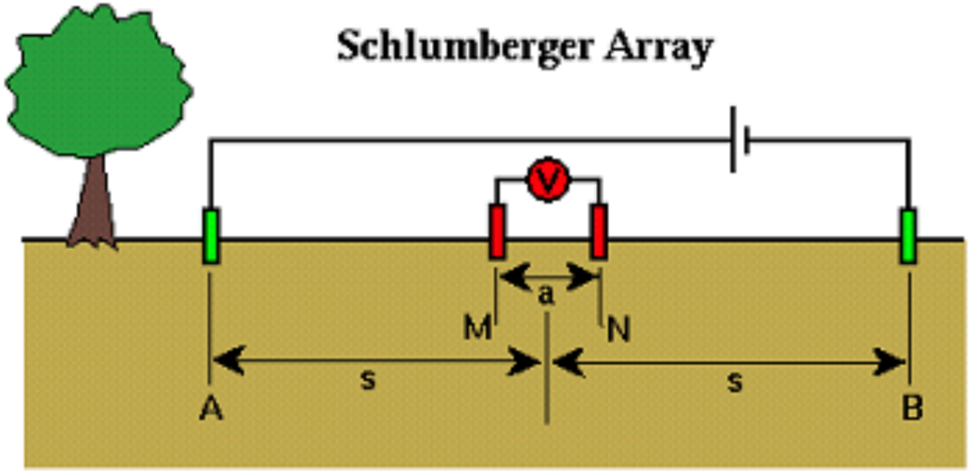


$$\rho_a = 2\pi a \frac{\Delta V}{i}$$

Wenner sounding data is plotted as apparent resistivity vs. a on a log-log plot

DC Resistivity Measurement

Schlumberger Array



$$\rho_a = \frac{\pi(s^2 - a^2/4)}{a}\frac{\Delta V}{i}$$

Schlumberger sounding data is plotted as apparent resistivity vs. s or AB/2 on a log-log plot

DC Resistivity Measurement

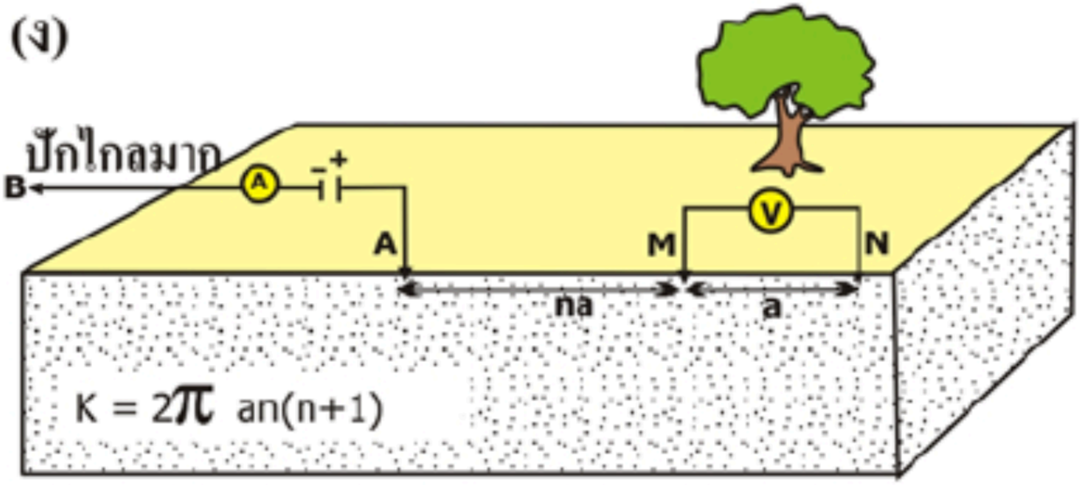
Dipole-Dipole Array

$$\rho_A = \frac{V}{I}\pi\, a\, n(n+1)(n+2).$$

Dipole-Dipole sounding data is plotted as apparent resistivity vs. *n a* on a log-log plot

DC Resistivity Measurement

Pole-Dipole Array



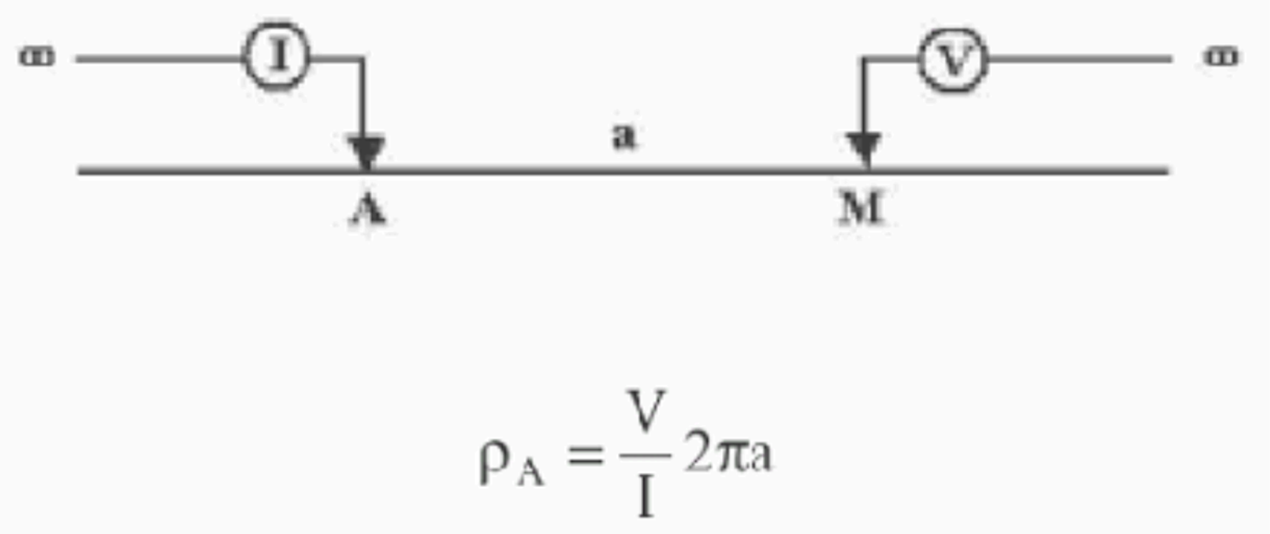
โพล-ไดโพล (Pole-dipole array)

$$\rho_a = 2\pi a n(n+1)\left(\frac{\Delta V}{I}\right)$$

Pole-Dipole sounding data is plotted as apparent resistivity vs. na

DC Resistivity Measurement

Pole-Pole Array



$$\rho_A = \frac{V}{I} 2\pi a$$

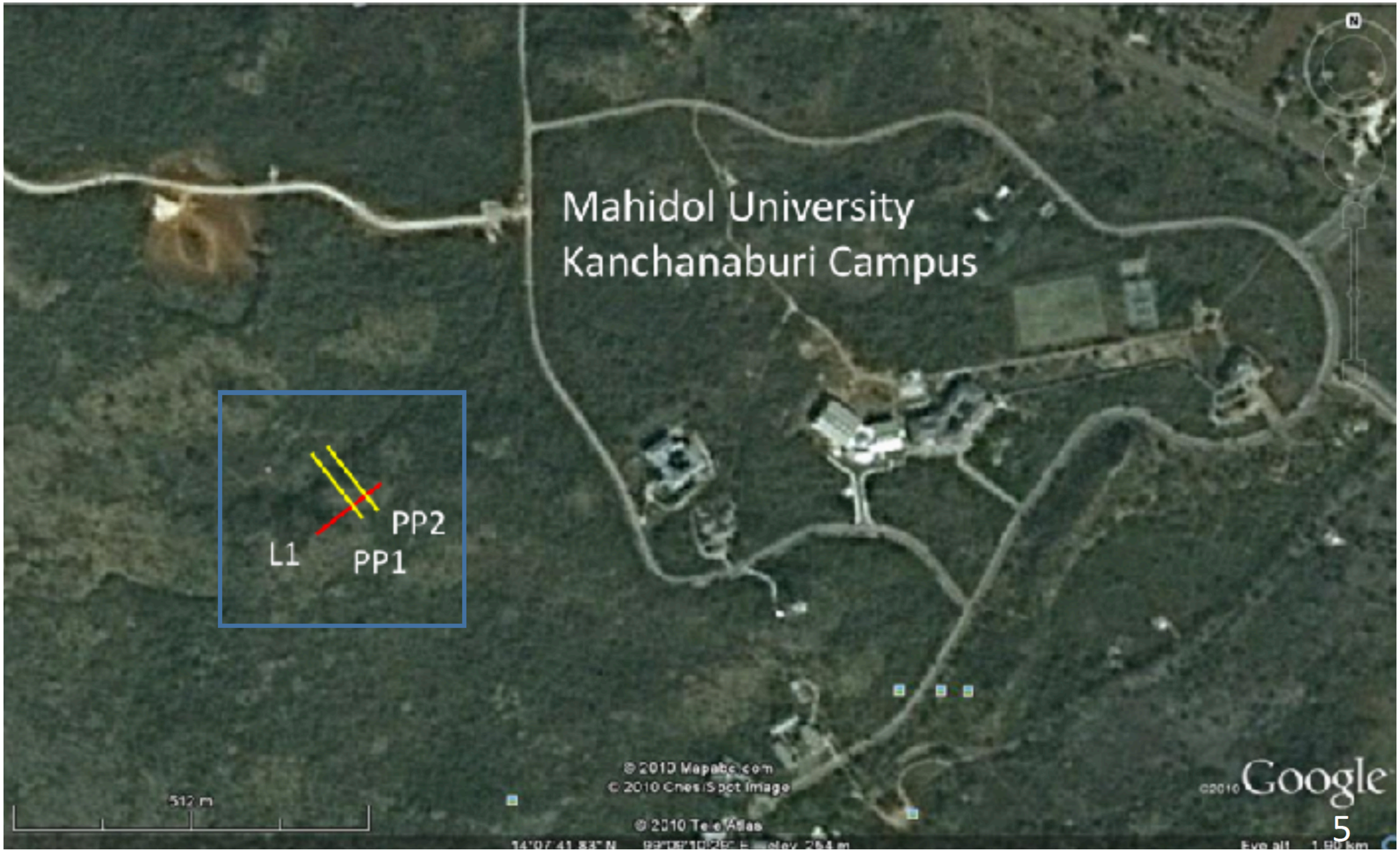
DC Resistivity Measurement

Array advantages and disadvantages

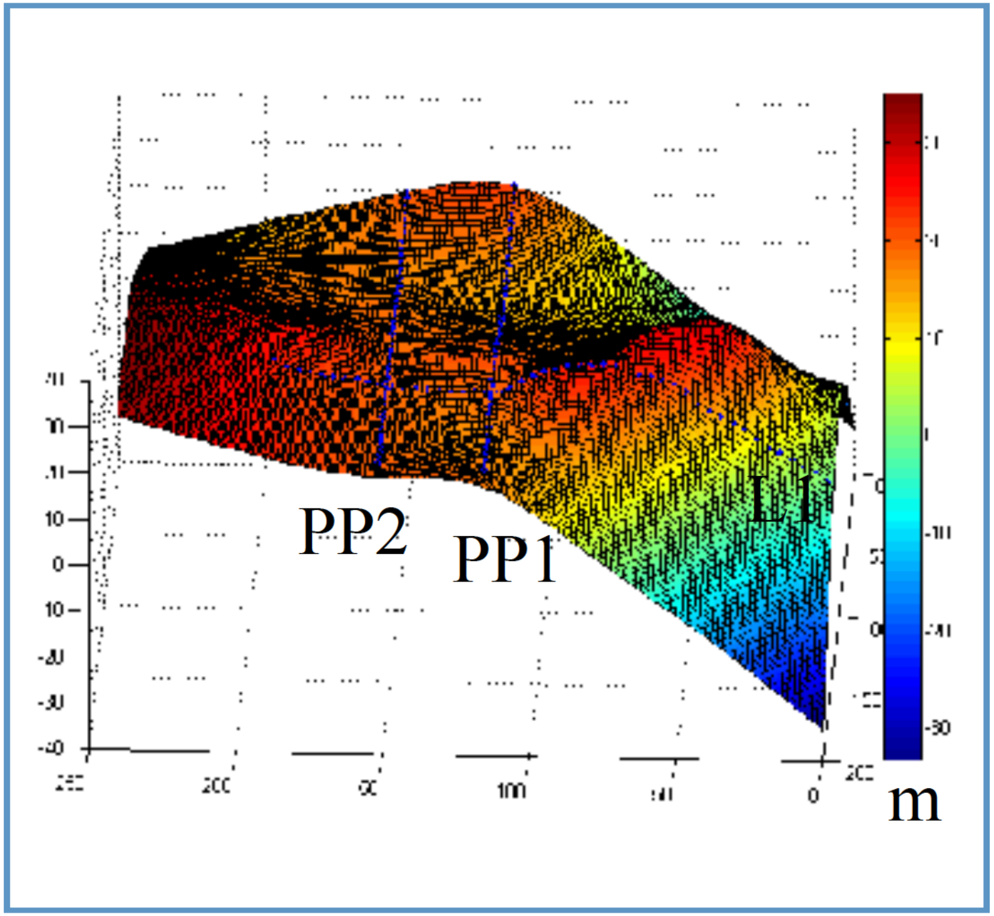
Array	Advantages	Disadvantages
Wenner	1. Easy to calculate ρ_a in the field 2. Less demand on instrument sensivity	1. All electrodes moved each sounding 2. Sensitive to local shallow variations 3. Long cables for large depths
Schlumberger	1. Fewer electrodes to move each sounding 2. Needs shorter potential cables	1. Can be confusing in the field 2. Requires more sensitive equipment 3. Long Current cables
Dipole-Dipole	1. Cables can be shorter for deep soundings	1. Requires large current 2. Requires sensitive instruments

DC Resistivity Measurement

Field Test: Study Area

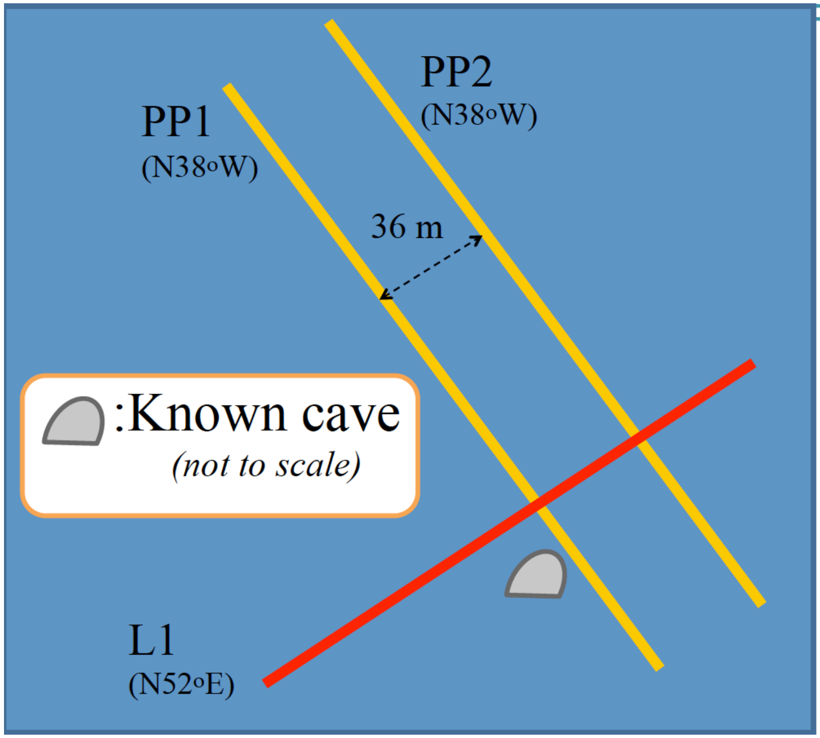


DC Resistivity Measurement



DC Resistivity Measurement

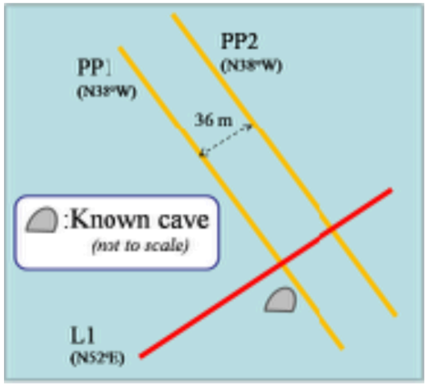
Field Test: Field Parameters

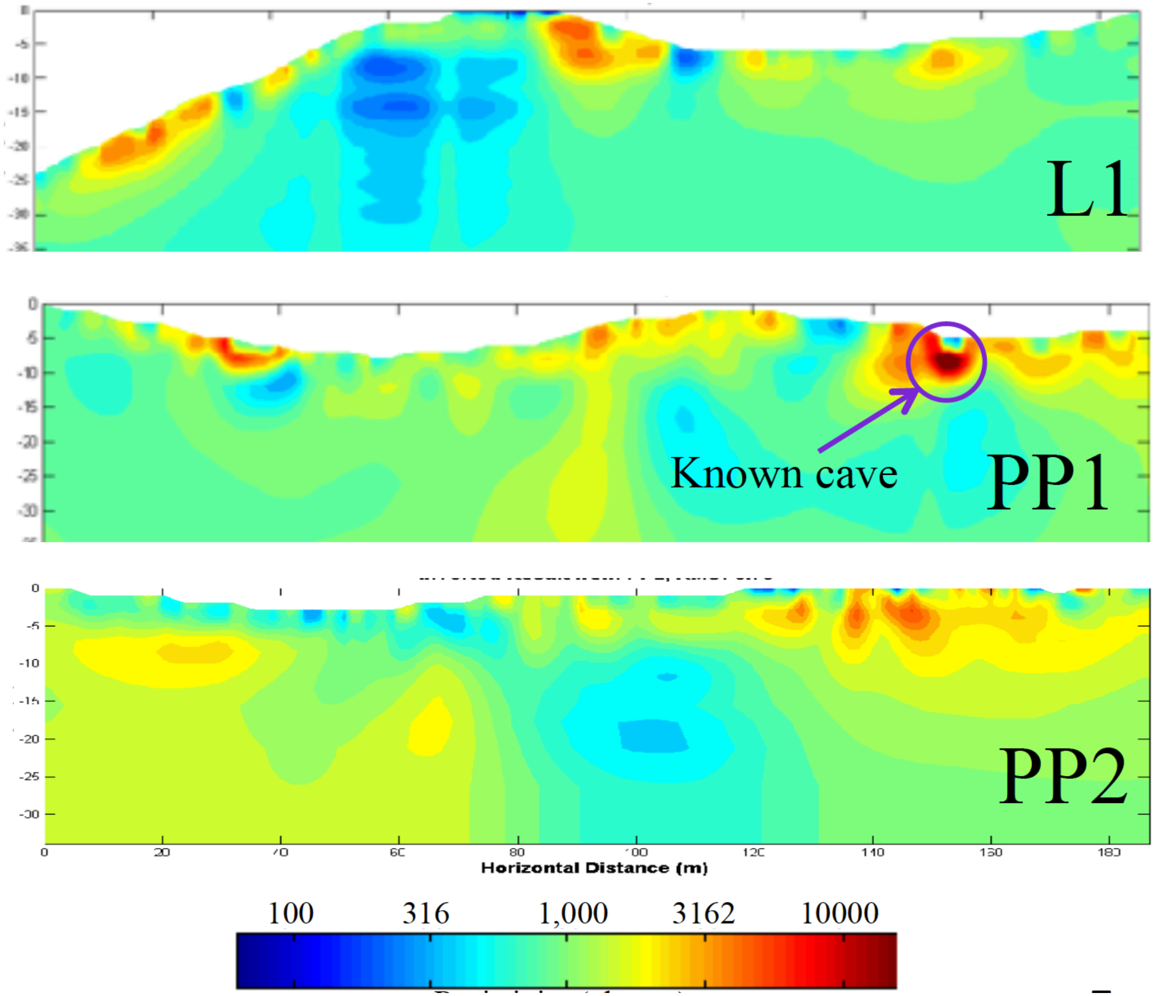


- Array: Schlumberger
- Profile length: 188 m
- Electrode spacing: 4 m
- Iris Instrument Syscal-R1 plus: 48 channels

DC Resistivity Measurement

Inverted Resistivity Models

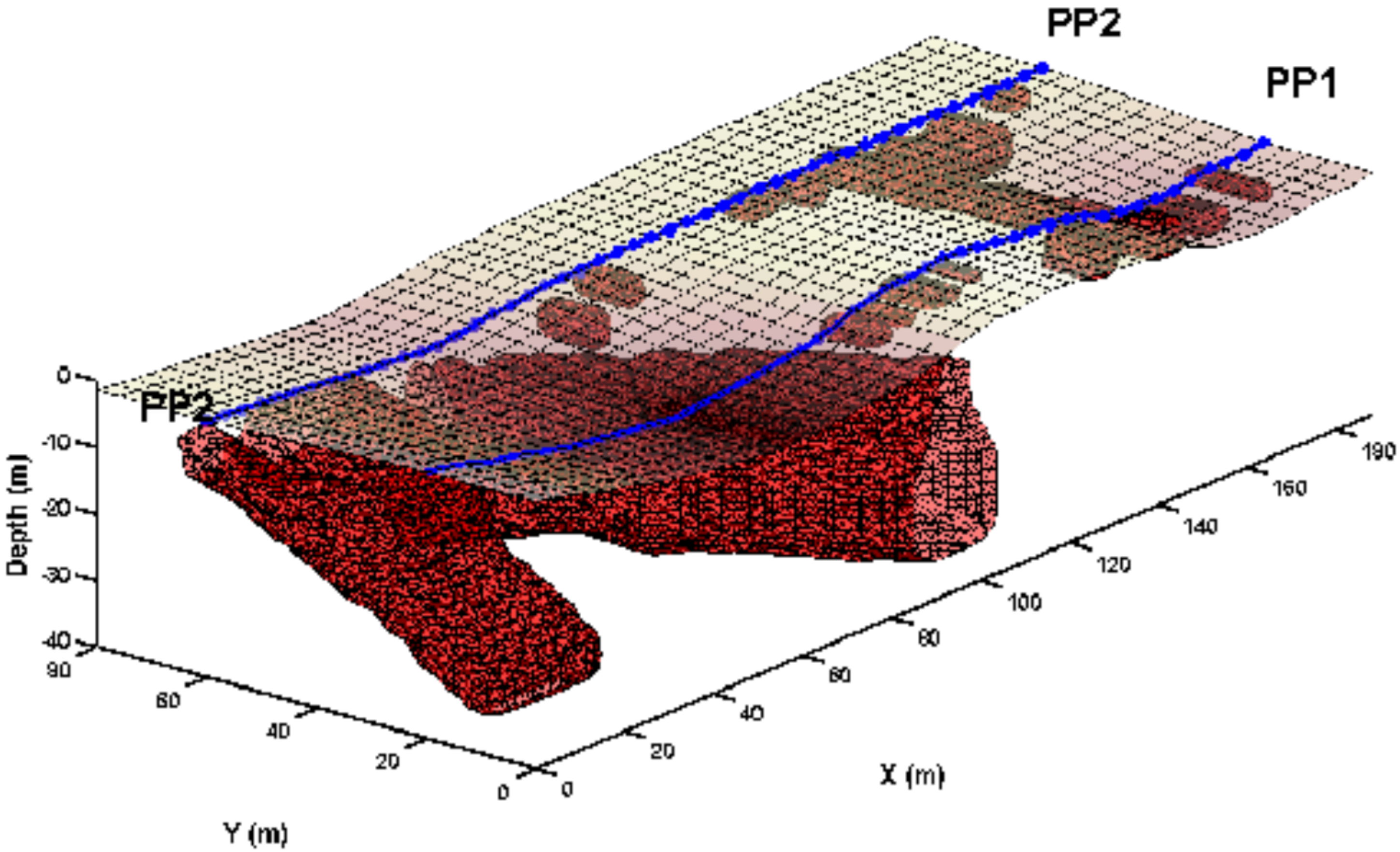




Inversion:
WSDCRInv-TH
(Vachiratienchai and Siripunvaraporn, 2011)

DC Resistivity Measurement

- 3-D : ERT (series of 2-D ERTs)



DC Resistivity Survey

Homework 1 Derive the apparent resistivity from the general expression:

$$\Delta V = \frac{I\rho}{2\pi}\left[\frac{1}{r_1} - \frac{1}{r_2} - \frac{1}{r_3} + \frac{1}{r_4}\right]$$

for these electrode configurations
1.1 Wenner array, 1.2 Schlumberger array, and 1.3 Dipole-dipole array

DC Resistivity Survey

Homework 2

Design the DCR experiment to find the ground water table of a depth of 20 m of a thickness of 5 m.